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A Vindication of Kantian Euclidean Space

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ABSTRACT

This paper is a phenomenological vindication of the Kantian claim concerning the Euclidean character of outer experience. The generally accepted view that the Kantian conception of space has been refuted either by the development of non-Euclidean formal geometries or by their empirical application within the framework of physical theories will be challenged. The significance of the Kantian claim for the modern Philosophy of Science will be expounded at the end of the text.

RESUMEN

Este artículo es una vindicación fenomenológica de la tesis kantiana sobre el carácter euclidiano de la experiencia externa. Se combatirá aquí la opinión, comúnmente aceptada, de que la concepción kantiana del espacio ha sido refutada por el desarrollo de las geometrías formales no euclidianas o por la aplicación de estas geometrías en el marco de ciertas teorías físicas. Al final del texto se expondrá la relevancia de la tesis kantiana para la actual filosofía de la ciencia.

I. THE STANDARD COMPLAINT AGAINST THE KANTIAN VIEW OF EUCLIDEAN GEOMETRY

When considering the validity of Kant's transcendental view of Euclidean space, one widespread complaint considers that Kant failed to distinguish between pure and applied geometry the way we do today. Pure geometry, as Hilbert showed, is a mere mathematical multiplicity, an axiomatic system interwoven by means of formal relationships where a priori intuition plays no role at all. Its claims have no empirical content whatsoever. Applied geometry, on the other hand, as exemplified by the use of non-Euclidean geometries by Einstein, has to do with the application of an abstract geometrical structure as a means of depicting the empirical world. This application is done under certain theoretical assumptions and the postulation of an empirical spatial congruence. Once the coordination of the geometrical structure with the empirical phenomena is established, it can be empirically tested. Euclidean geometry seems just one formal "mathematical structure" among others, whose correspondence with the physical world is not imposed. There is then no place for the idea that Euclidean geometry is constitutive of all possible experience. The transcendental a priori validity of Euclidean geometry, as implicitly ascertained by Kant in the mathematical principles of the pure understanding appears, thus, to have been refuted.

The reasons summarized above do certainly look very compelling. However, I want to argue that these reasons do not at all refute what Kant actually stated when he claimed the necessary character of Euclidean geometry for all possible outer experience. Before carrying out my vindication, in the next section I will consider in which terms Kant actually claimed the a priori character of Euclidean geometry. I will establish that the underlying sensible character of Kant's transcendental position must be taken into account in order not to misrepresent his thought. In the following sections I will then show that Kant's transcendental conception of Euclidean space is not at all challenged by the reasons previously mentioned. Furthermore, I will hold that the a priory character of Euclidean space imposes on us with an immediate phenomenal legitimacy.

II. The Sensible Character of the Kantian Claim

The principles of pure understanding express conditions and necessary determinations of every object given in a possible experience. To understand what this really means, it is convenient to keep in mind what the "possibility of experience" means for Kant. "Possibility" is used by Kant in the medieval sense attached to the term "possibilitas", meaning the essence, *quidditas*, form or nature of something. The term "experience" refers to what is empirically given but only inasmuch as it is considered in its *objective* character, that is, as something opposing us which can be objectively determined through judgements. The possibility of experience is, therefore, the very essence of that which constitutes empirically objective reality or, in other words, "that which gives all of our cognitions a priori objective reality" [Kant (1781), A 156].

It is well known that, for Kant, what is sensibly given is to be articulated under forms of unitary syntheses in order to appear as experience. Experience is thus conceived by Kant as the product of objective syntheses *of what is given in sensibility.* The categories are the pure or a priori forms of syntheses that conform the possibility of experience, that is, the essence of an unitary objective apperception. The synthetic unity of experience is, therefore, "that of the combination of the manifold of a given intuition in general in an original consciousness, in agreement with the categories, and applied only to our sensible intuition" [Kant (1787), B 161].

While keeping in mind the sensible root of the syntheses which make experience possible, let us now consider on what ground claimed Kant the necessity of the propositions of geometry in experience. For Kant, space and time are a priori forms of our sensibility. The propositions of Euclidean geometry and those of mathematics in general are a priori because they are "constructed" on these a priori forms of sensibility, that is, they do not rest on particular empirical sensations but are made possible solely by means of an objective syntheses on pure intuition: "all geometrical cognition is immediately evident because it is grounded on intuition a priori, and the objects are given through the cognition itself a priori in intuition" [Kant (1781), A 87]. Furthermore, what the geometrical propositions state a priori hold necessarily for *all possible experience* because the sensible form under which geometrical propositions are constructed is set a priori in experience, and because the same synthetic forms of unity which make up the objectivity of geometrical propositions are the ones which pre-configure the objective apperception of experience:

> The syntheses of spaces and times, as the essential form of all intuition, is that which at the same time makes possible the apprehension of the appearance, thus every outer experience, consequently also all cognition of its objects, and what mathematics in its pure use proves about the former is also necessarily valid for the latter. [Kant (1781), A 165].

It is then clear from what has been said, that only in relation to a *sensible experience* could a priori validity of Euclidean geometry be claimed by Kant. However, when Kant's transcendental conception of Euclidean geometry is allegedly refuted making appeal to the development of formal non-Euclidean "geometries" or to their use within certain physical theories, the sensible horizon of the Kantian conception is laid apart. We will now see why this conception is not at all menaced by this type of challenges.

III. The Alleged Challenges Posed by the Theory of Relativity

Let us consider first the claim that the theory of relativity is an empirical refutation of the Kantian doctrine concerning the a priori Euclidean character of space.

A geometry gains empirical relevance only after the postulation of certain physical invariants is assumed within the physical theory. In other words, before claiming that a particular geometry holds for the physical world it is always necessary to state, among other things, which are the elements that are taken as empirically invariant within the physical theory, that is, which are taken as physically irreducible. In the theory of relativity, for instance, absolute criteria of spatial congruence have to be postulated for "rigid bodies". These criteria are not introduced at random since they are guided by certain regulative principles like the simplicity and homogeneity of the physical theory they help to build, but they are, certainly, neither empirically descriptive nor "absolute", in the sense that these criteria have to be postulated or introduced "a priori". A consequence of this is that physically applied geometries cannot be merely qualified as empirically true or false. In the words of Max Jammer:

> It is a matter of convention which geometry we adopt, but only as long as no assumptions are made concerning the behaviour of physical bodies as implied in the measurements. Once these assumptions are laid down, the choice of the geometric system is determined. As Einstein explains, it is the sum total of the assumptions of correlation and of the system of abstract geometry that has to conform to experience. Once the principle that relates rigid bodies to Euclidean solids is accepted, it is experience that conditions the choice of geometry [...]. Hence it is clear that the structure of the space of physics is not, in the last analysis, anything given in nature or independent of human thought. It is a function of our conceptual scheme [Jammer (1993), pp.172-173].

Since physically applied geometries are to be taken as empirically adequate only after a certain "coordinative framework" has been theoretically established, physical theories with the same empirical adequacy and predictive power may give birth to different geometrical accounts of the world. The decision to choose between them cannot be made in terms of mere empirical adequacy. This relative character of the physical adequacy of geometry is, to a great extent, generally recognized.[1]

If we turn now to consider how Kant understands the a priori validity of Euclidean geometry in experience, it is easy to realize that his claim has little to do with the empirical adequacy of a geometrical structure in the framework of a physical theory. When Kant says, for instance, that the shorter line between two points is necessarily a straight line, he does not, of course, intend to express an empirical relationship verifiable through a certain procedure of measurement under certain physical assumptions. Kantian geometrical propositions do not point at empirical relations dependent on empirical objects, but have an "immediate" character: space is here "to be regarded as the condition of the possibility of appearances, not as a determination dependent on them… Space is not a discursive or, as is said, general concept of relations of things in general, but a pure intuition" [Kant (1781), A 24]. The empirical adequacy of a geometry, established through measurements under the assumption of a certain theoretical framework has, therefore, nothing to say in what concerns the a priori Euclidean character of experience which Kant claims.

IV. The Alleged Challenges Posed by the Mathematical Development of non-Euclidean Geometries

Let us turn now to consider non-Euclidean geometries to see whether their mathematical development may counter the Kantian claim in some sense.

It has been shown earlier, that only inasmuch as geometrical propositions are built on sensibility did Kant safeguard their a priori validity in experience. Kant states that all mathematics would be empty fictions if void of their sensible character, they would be nothing but a set of formal relationships with no direct relation with what is given in sensible outer experience [Kant (1781), A 160; Kant (1783), p. 287]. This is what effectively happens when "geometries" are constructed as implicit definitions void of all intuitive content as in Hilbert's axiomatization of geometry. "Geometries" of this sort are just formal mathematical structures that operate with rules. Neither a correspondence between their primitive concepts and intuition nor a depiction of a sensible experience is intended here. Surely, such formal mathematical structures can find a model, a "physical application" in the empirical world but, as we have discussed before, their empirical adequacy is not immediate but can only be built under certain theoretical assumptions.

Since formal geometries have no immediate sensible significance, the development of non-Euclidean formal geometries neither supports nor hinders the plausibility of the Kantian claim concerning the a priori Euclidean character of experience. On the other hand, it can well be said that the only thing proven by the development of non-contradictory non-Euclidean formal "geometries" is the synthetic character of Euclidean geometry. Something which Kant himself always sustained.

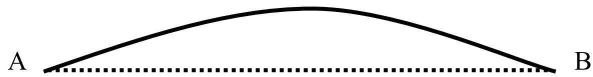
V. Phenomenological Vindication of the A Priori Euclidean Character of our Sensible Experience

We should then distinguish among three different meanings attached to the concept "geometry". By "geometry" we can first understand a formal mathematical structure emptied or detached of any kind of correspondence with phenomenal space; second, an empirically adequate mathematical structure whose empirical adequacy, however, is relative to a set of pre-established theoretical and physical assumptions; and third, the character of space in a "sensible" or "immediate" phenomenal sense. I have argued that the Kantian claim concerning the necessity of Euclidean geometry is to be taken in this third sense only, since he conceived geometrical propositions only inasmuch they are rooted in pure intuition.[2]

However, even if our previous discussions may have been helpful to clarify what is the point at stake with the Kantian claim concerning the a priori Euclidean character of outer experience, they have done nothing to support the plausibility of that claim. The real difficulty of Kant's transcendental conception of Euclidean geometry arises now: if we have mathematically proven that formal geometrical structures different from the Euclidean one are possible, on what grounds are we to accept the claim that Euclidean geometry is valid a priori for all possible experience? Why are we to assume that our immediate experience has a Euclidean configuration? Why are we to assume that outer experience, in a Kantian sense, cannot have a non-Euclidean structure? Let us now consider this problem.

It is certainly true that anyone can have, to a certain degree, a grasp of non-Euclidean geometries. For example, we can visualize a curved surface and think of it as the space where a spherical or pseudo-spherical geometry take place. However, we have to take into consideration that this sort of "representations" of non-Euclidean geometries are not immediate. In order to "represent" a space with a non-Euclidean character we have to "conceive" a certain underlying metric that turns the immediately given space "only apparent". In the example given, we are thus still lacking an immediate empirical representation of a "spherical space". But this point needs to be clarified in more detail.

Let us guide our discussion with a simple example and consider whether the shorter line between two points is or is not necessarily straight in a Euclidean sense. If we take this relationship in an immediately "intuitive" sense I have no doubt that the answer can only be affirmative. This has, however, been challenged by some critics of the Kantian conception of space like von Helmholtz and Hans Reichenbach. These critics argue that in the same way that we visualize Euclidean space, we can also visualize non-Euclidean spatial structures. The only reason we give a privileged status to Euclidean space, they say, is habit, an empirical reinforcement of the relations we are used to encounter in objects given in everyday experience [Von Helmholtz (1968), p. 132; Reichenbach (1958), p. 57]. In other words, it is argued that if we progressively got used to a spatial metric different from the Euclidean one, we could intuitively deal with non-Euclidean spaces and have a grasp of non-Euclidean spatial relationships with the same immediacy we assume the Euclidean character of space in our everyday life. The tendency to attribute a Euclidean character to our immediate spatial experience would then depend on the underlying assumption that a Euclidean spatial metric reigns in space. This assumption, Reichenbach argues, is not a priori or necessary but one that could be empirically reinforced and progressively modified [Reichenbach (1958), pp. 48-58]. Myself, I do not understand how an immediate empirical visualization of non-Euclidean geometries is to be made possible. While I am sympathetic with the idea that a readjustment of the immediate grasp of the spatial metrical relationships is possible, I disagree with Reichenbach's idea that this may have anything to do with a refutation of the Kantian claim concerning the a priori validity of Euclidean space in our immediate outer experience. The question is not which type of metric we assume when ascertaining spatial relationships or whether we can get used to a spatial metric different from the Euclidean one. The question is whether or not we can make sense of an inherent character of our immediate outer experience *before* any metrical assumption is taken, and whether or not we can ascertain its Euclidean character. I believe we can. Let me now explain this with a rough example. Assume that the continuous line in the picture between points A and B is the projection onto a plane of a straight line in a non-Euclidean Riemannian "space":



Now, the dotted line would represent, within that Riemannian spatial metric, a longer line than the continuous one. I do not deny that we could get used, through habit, to a Riemannian metric to the point of identifying the dotted line as "physically" longer than the continuous one. What I reject is that this may have anything to do with a sensible "intuition" of Riemannian space, as if, through an empirically reinforced habit, we had reconfigured our immediate visual experience. We may also say that we can come to a certain grasp of Riemannian space, but only in the sense that we can end up assuming, through habit, certain underlying empirical relationships which establish the type of metric that reign in experience. This grasp, however, would not have the immediacy of a "sensible" experience. Now, the question to be posed in our example is this: does the immediate apprehension of the dotted line as being shorter than the continuous one presuppose, in the same way, the previous assumption of a Euclidean metric? It seems to me this is only so if the relationship "being shorter than" is understood as holding among physical objects: within the horizon of an immediate sensible experience and prior to the assumption of certain physical relationships or metric, it is immediately imposed on us that the continuous line is longer than the dotted one. We come to apprehend this relationship phenomenally, by means of an immediate recourse to "intuition" (of course, here I mean *intuition* not in a Kantian but in a general phenomenological sense). And, precisely, the point at stake here is that non-Euclidean spatial relationships can never be "intuited" in this same way. To put it plainly: our immediate spatial experience has, as such, a Euclidean configuration.

Let us come back now for a moment to Kant's thought. Kant held that the grasp of geometrical relationships required a sensible synthesis to be objectively apperceived. The Kantian syntheses of apprehension, reproduction, and recognition are necessary conditions for a sensible object to be given. Only through these syntheses, if we follow Kant, can the lines of our previous example be opposed to us as sensible objects of experience. Now, we can realize that if we were accustomed to a Riemannian metric, the character of what is empirically apperceived would not be modified: in an immediate experience, the sensible lines of our example *are the same* whether we imagine them articulated within the horizon of a Euclidean or a non-Euclidean physical metric. It is then obvious that the Kantian a priori synthesis of apperception has nothing to do with the assumption of a particular metric. In other words, the a priori sensible synthesis is linked with an empirical immediacy which is foreign and independent of the assumption of a particular metric in physical space. Even if we accept that a certain metric may end up, through habit, being very much like a "*pre*-conception" of the spatial relationships holding in experience, it must be however recognized, that this empirically reinforced "*pre-*" is not the one Kant deals with when he talks of the *a priori* sensible synthesis which conforms our experience. Kantian sensible a priori syntheses of the transcendental imagination is a previous necessary condition for the assumption of a certain metric and cannot be empirically reinforced or modified the way the assumption of a metric can. Now, the important thing to notice here is that the Euclidean character of our immediate experience holds as soon as the objects are sensibly apperceived, that is, this character is already into force in every immediate experience before any relational metric is assumed. Euclidean geometrical features can therefore be taken to be inherent to any objective apperception. Or, in a Kantian sense, it can be said that this features configure the possibility of outer experience.

In order to vindicate that, in a Kantian sense, there is only one straight line between two points, that this straight line is the shortest of all possible lines or that two parallel lines never touch one another, we have not appealed to formal axioms nor to a empirical adequacy gained having recourse to certain physical postulates. Only in an "intuitive" cognition can we justify the validity of these geometrical relationships. In other words, the Euclidean character of experience claimed by Kant can only be justified *phenomenally*. This kind of immediate phenomenal evidence is the only means we can use to vindicate or refute the Kantian claim concerning the a priori character of Euclidean geometry because, as we explained before, only within the realm of an immediate experience did Kant make it.

VI. Euclidean Space and the Philosophy of Science

My vindication of Kant finishes here. It must be now pointed out that Kant mistakenly thought that the constitutive conditions of all possible experience hold also as necessary features of the natural sciences, claiming therefore that "the principles of possible experience are at the same time universal laws of nature, which can be known a priori. And thus the problem […] 'how is pure science of nature possible?' is solved" [Kant (1783)]. According to Kant, Euclidean geometry would be normative not only in experience, but also within the theoretical framework of the natural sciences. With this claim, which has been factually refuted, Kant went beyond what he had actually shown in the *Critique of Pure Reason*. Taking into account the dominance of Newton physics during his lifetime, it is quite understandable why Kant exceeded the limits of his own achievements. Since the necessity of Euclidean geometry is linked with its sensible character, its a priori validity has to be limited to what is immediately given in experience. This limitation prevents us from mistakenly assuming that the use of Euclidean geometry is necessary within empirically adequate physical theories. This limitation, however, does not block consideration of the implications of the Kantian claim for a philosophical reflection of the Natural Sciences.

While it is normally admitted that any theoretical determination of experience is framed within the assumption of certain principles, it is usually denied that these can be normative a priori. With respect to Kant´s thought it is thus widely held that "we must retain Kant's characteristic understanding of a priori principles as constitutive […] while rejecting the more traditional marks of necessity, unrevisability, and apodictic certainty" [Friedmann (1999), p.73]. Myself, I also agree that the assumption of certain principles is constitutive of modern physical science in the sense that theoretical projections are inherent to the constitution of physical theories. But I strongly oppose the suggestion that a priori necessity has no role at all to play when reflecting on scientific theories. The normative character of Euclidean geometry is certainly not legitimate when claimed beyond an immediate experience but it is to be saved when restricted to it. Now, if we are to ask what is the relevance of this normativity for the philosophy of science we have to ask what role our immediate sensible experience plays for the constitution of our physical theories and what role should its a priori configuration plays in the philosophical reflection of the empirical sciences.

It is important not to forget that immediate sensible experience is there "into force" before any empirical scientific theory is built. Furthermore, empirical knowledge ends up referring, in one way or another, to this immediate sensible experience. In other words, even if empirical scientific theories aim at general and intersubjective determinations, the immediate sensible experience is always the phenomenal ground to which they must end up referring to. Certainly, in order to accomplish their goals, empirical theories have to transcend the immediacy of experience through the institution of certain theoretical assumptions, however, only referring to an immediate sensible experience can *empirical* scientific theories, inasmuch as empiric, be constituted. And thus, standing on the ground of an immediate experience with a Euclidean structure we end up with physical theories, void of immediate empirical content that depicts a non-Euclidean structure of the world. Now, the relevance of the Kantian claim does not lie in vindicating a privilege of Euclidean geometry when used within physical theories but in helping us to put into context the real character and meaning of what physical theories ultimately say. Since physical theories are built up having recourse to an immediate sensible experience we should not be willing to give blind ontological credit to whatever claim about the spatial structure of the world comes from a physical theory. The gap between the structure of our immediate experience and what is claimed within a physical theory should lead us to identify what are the theoretical projections and assumptions that make the latter possible and be also aware of the true significance and limits of the claims there stated.

What I mean here can be put in connection with the Husserlian notion of the Lifeworld. The Lifeworld can be understood as that which is always and already valid for us before any scientific theory is constituted. It is that which is "in force" as immediately given before any scientific theory is built, the ground on which any empirical science has to stand and the original phenomenal foundation of theoretical scientific constructions. If we take the Husserlian notion of the Lifeworld in this limited sense, the Euclidean structure of our sensible experience claimed by Kant can be exactly conceived as an essential feature of our Lifeworld. And this is a feature of that phenomenal point of departure on which the constitution of any elaborate empirical theory depends. In reflecting about the ultimate significance and relevance of our empirical scientific achievements this dependence should always be kept in mind.

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NOTES

[1] Classical expositions of this position can be found in Von Helmholtz, H. (1968), pp.130-131; Poincaré, H. (1953), especially, p. 180; Einstein, A. (1953), especially, p.192; Reichenbach, H. (1958), pp.14-19, 30-37.

[2] "All mathematical cognition has this peculiarity: it must first exhibit its concept in intuition and indeed a priori; therefore in an intuition which is not empirical but pure. Without this mathematics cannot take a single step; hence its judgements are always intuitive". [Kant (1783), p. 287].

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