

Infinite Regresses, Infinite Beliefs

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One way of mapping part of the domain of epistemology is to represent various theories as responses to the following argument:

- (I) A belief (strictly, a token state of belief) is justified only if a justified belief is a reason for it. (Premiss.)
- (II) There are justified beliefs. (Premiss.)
- (III) The proper ancestral of the reason-relation is irreflexive. (Premiss.)
- (IV) There is an infinite sequence (strictly, a sequence with infinite range) of justified beliefs each of which is a reason for its predecessor, if any. (From (I) to (III).)
- (V) There is no such sequence. (Premiss.)
- (VI) There both is and is not such a sequence. (From (IV) and (V).)
- (VII) Not-(I) / not-(II) / not-(III) / not-(V). (Reductio.)

The argument instantiates a schema for infinite regress arguments generally:

- (i) $(\forall x_1)[Ax_1 \rightarrow (\exists x_2)(Ax_2 \ \& \ x_1Rx_2)]$. (Premiss of generation.)
- (ii) $(\exists x_1)(Ax_1)$. (Premiss of existence.)
- (iii) *R/R is irreflexive. (Premiss of linearity.)
- (iv) $(\exists s)[\text{Inf}(R(s)) \ \& \ (\forall i)(i \in D(s) \rightarrow As_i \ \& \ As_{i+1} \ \& \ s_iRs_{i+1})]$. (From (i)-(iii).)
- (v) \sim (iv). (Premiss of finitude.)
- (vi) (iv) & \sim (iv). (From (iv) and (v).)
- (vii) \sim (i) / \sim (ii) / \sim (iii) / \sim (v). (RAA.)

The deduction of (iv) from (i) to (iii) is fairly straightforward (Black 1996, 102-103): first, an inductive procedure is specified, using the principle of dependent choice, for generating from (i)-(iii) a sequence that satisfies the second conjunct of (iv); second, the range of the sequence is proved to be infinite; third, (iv) is inferred with the rule for introducing the existential quantifier. Formerly I used two premisses in place of (iii), that (viii) R is irreflexive and (ix) R is transitive, to derive (iv) (Black 1987a, 404-405); the present formulation is more frugal because (iii) is true if, but not only if, (viii) and (ix) are true.

In these terms the problem of infinite regresses of justification is this: which premiss in the argument is false and hence should be rejected at (VII)? If (I) is false, a component of a form of foundationalism is true: i.e. there are beliefs that are basic in the strong sense that they are justified even if they do not have reasons (this claim is supported by certain theories of privileged access and certain externalist theories of justification); or in the weaker sense that they are justified even if they do not have reasons that are themselves beliefs (this is supported by the view that a sense-experience can be a reason for a belief); or in the still weaker sense that they are justified even if they do not have reasons that are beliefs that are in

turn justified (this is supported by certain contextualist theories of justification). An unjustified belief might itself be called basic where it is a reason for some justified belief. These claims constitute only parts of foundationalist theories of justification; a full theory will also state that, and how, justified non-basic beliefs derive their justification from a relation to basic beliefs. The account is likely to be in recursive terms. Note that the *truth* of (I) is compatible with claims made by other forms of foundationalism, e.g. that some justified belief is basic in the sense that - contrary to (III) - it is a reason for itself. Note also that, if (I) is construed as a material conditional, (II) is true if (I) is false.

Arguments like this are often used to derive a foundationalist conclusion, but there are other possibilities. If (II) is false, a radical form of scepticism is true: there are no justified beliefs. If (III) is false, there are circles of reasons; i.e. there is either (a) a finite sequence of beliefs each of which is a reason for its predecessor, if any, and the first is a reason for the last or (b) - the limit case already noted - a belief that is a reason for itself. If in addition all the other premisses are true, a form of coherentism is true; i.e. a justified belief is justified by virtue of being one of a set of beliefs that form a circle of reasons.

If (V) is false, there is an infinite sequence of the kind whose existence it denies: call them J-sequences. Of the premisses (V) is the least likely to be rejected in the conclusion, first, because that would make the last three steps of the argument redundant - not-(V) is equivalent to (IV) - and second because (V) is highly plausible. I believe that (V) is true, but it has sometimes been defended by bad arguments. I shall discuss two such defences.

Assume that the reason-relation connects beliefs held by the same person (this follows from (DR) below). Then the discussion of (V) can be broken into three questions:

- (Q1) Does there exist an infinite sequence of beliefs held by the same person?
- (Q2) Supposing that such sequences exist, does any of them comprise only justified beliefs?
- (Q3) If so, is any of them a J-sequence?

(V) is true iff the answer to any of these is no. The answer to (Q1) is yes, provided "beliefs" embraces dispositional as well as occurrent beliefs. Let S^P be the infinite sequence of propositions "2 > 1", "3 > 1" etc. I believe each of these, so there is a corresponding infinite sequence S^B such that each S^B_i is a belief of mine whose object is S^P_i . Clearly there are some elements of S^P that I never consider: the elements of S^B corresponding to these are dispositional.

To this it has been objected that at some stage in S^P there is a proposition S^P_n that cannot be the object of a belief, occurrent or dispositional, because the number it refers to is too large to consider (Williams 1981, 86). This objection is the first bad defence of (V). Let it be granted that a proposition cannot be believed if it refers to a number too large to consider: this cannot be used to show that I do not believe each element of S^P , for no proposition in S^P refers to such a number. All the numbers referred to by these propositions are natural numbers; but since

Cantor mathematicians have been considering numbers - the alephs - bigger than any natural number. If a number has been considered, it is not too large to consider; and, if a number is not too large to consider, no number smaller than it is too large to consider. So none of the numbers referred to by elements of S^P is too large to consider. Someone who thinks that transfinite arithmetic is misconceived may reply that, although mathematicians have performed operations with the symbols " \aleph_0 ", " \aleph_1 " etc, they have not been considering numbers. But, even if that is right, the answer to (Q1) can be supported by an example that raises no qualms about the infinite: the sequence of propositions " $1 \frac{1}{2} > 1$ ", " $1 \frac{2}{3} > 1$ ", etc.

A second objection (implicit in Williams 1981, 86), which constitutes the second bad defence of (V), runs thus: (a) there are natural numbers too large to write down or express in any other way; (b) among the elements of S^P are propositions that refer to such numbers; (c) likewise, therefore, these propositions are inexpressible; (d) but a proposition is believed only if it can be expressed; so (e) no one believes all the elements of S^P . (d) is doubtful, but the argument anyway fails at (a). It is true that there are natural numbers that it is humanly impossible to write down in Arabic notation, but these can be expressed with abbreviations. It might be replied that, whatever list of abbreviations is employed, there will be natural numbers too large to express with them. But this can be conceded; for to reach (a) the quantifiers in the reply need to be reversed, yielding the implausible claim that there are natural numbers too large to express with any list of abbreviations.

The answer yes to (Q1) therefore stands. S^B also establishes that answer to (Q2). Distinguish between an actional and a statal sense of "justified": roughly, a belief is actionally justified iff the believer has applied a procedure that justifies it, and statally justified iff he *can* apply such a procedure. I can apply such a procedure to each S^B_i ; it consists in an application of any standard set of axioms for number-theory. So S^B is an infinite sequence of justified beliefs held by the same person.

It follows that S^B establishes the answer yes to (Q3) iff each S^B_i is a reason for its predecessor if any. Whether this is so depends on the analysis of the reason-relation. There are various kinds of reason connected with beliefs: one strong definition is:

(DR) B1 is a reason for B2 iff there exist a person N and propositions P and Q such that (a) B1 is N's belief that-P, (b) B2 is N's belief that-Q, (c) P confirms Q and (d) B2 is based on B1.

The relation of basing in (d) can be roughly defined thus:

(DB) N's belief that-Q is based on his belief that-P iff the fact that N believes that-Q is explained by the fact that there is an appropriate causal chain from his belief that-P to his belief that-Q,

where "There is a causal chain from X to Y" means "X stands to Y in the proper ancestral of the relation ...causes...".

Given (DR), S^B is a J-sequence iff, first, each S^P_{i+1} confirms S^P_i and, second, S^B_i is based on S^B_{i+1} . It is implausible to hold that either of these requirements is met (Black 1987b, 178-181). I am unable to think of a better example of a J-sequence, and so conclude that there is no good case for the answer yes to (Q3). But nor is there a decisive case for the answer no. Someone might seek to establish that answer by invoking (DB) and arguing from the premiss that a dispositional belief cannot be a cause; or that an infinite sequence of causes is impossible, or would last too long; or that a sequence of causes and effects must include a first cause; or that there can be no infinite sequence of explanations. But no such argument seems to work (Black 1987b, 181-190; Black 1988, 436).

Since (V) is true iff the answer to any of (Q1)-(Q3) is no, and since the answer to (Q1) and (Q2) is yes, the intuitive plausibility of (V) tips the balance in favour of the answer no to (Q3); but that answer will have to be reversed if an example of a J-sequence can be found. Can you think of one?

Literature

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