Wittgenstein´s Critique of Gödel´s Incompleteness Results

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Motto: "Don´t treat your common sense like an umbrella. When you come into a room to philosophize, don´t leave it outside but bring it in with you" (Wittgenstein 1939 LFM Unit VI page 68)

It is often said that Gödel´s famous theorem of 1931 is equal to the Cretian Liar, who says that everything that he says is a lie. But Gödel´s result is only similar to this sophism and not equivalent to it. When mathematicians deal with Gödel´s theorem, then it is often the case that they become poetical or even emotional: some of them show a high esteem of it and others despise it. Wittgenstein sees the famous Liar as a useless language game which doesn´t excite anybody. Gödel´s first incompleteness theorem shows us that in mathematics there are puzzles which have no solution at all and therefore in mathematics one should be very careful when one chooses a puzzle on which one wants to work. Gödel´s second imcompleteness theorem deals with hidden contradictions – Wittgenstein shows a paradigmatic solution: he simply shrugs his shoulders on this problem and many mathematicians do so today as well. Wittgenstein says than Gödel´s results should not be treated as mathematical theorems, but as elements of the humanistic sciences. Wittgenstein sees them as something which should be worked on in a creative manner.

1. Gödel in 1931 worked near the abyss of contradiction

Let me first give a brief overview on Gödel´s results of 1931 – because when I speak of Gödel I am always only thinking of this year. Gödel has achieved many other results, but they are not discussed in literature to the same degree as his result of 1931.

Gödel has proved in a formal manner that mathematics has gaps. His result is very tricky. In secondary literature I have found the statement, that Gödel works near the abyss of contradiction (see my book on the Heptagon, Ohmacht 1997, page 88). I want to show this closeness to contradiction by exposing two statements to you, both of which bear a close resemblance to Gödel´s result, but the two are different in the sense that one renders a contradiction and the other renders the desired result, and this situation exists even though these two sentences are very similar in their structure.

The first statement was and always has been very popular within philosophy. The sentence is attributed to Epimenides. (Taschner (2002, page 17) maintains that it was reported by a Sophist named Eubulides). It has something puzzling in it. It runs as follows:

„I am lying“

A friend of mine, who is a computer specialist is always saying that Gödel´s result stemmed from this utterance, and he is right in the sense that Gödel was inspired by this sentence. One can read on page 149 of the first volume of Gödel´s “Collected Works”: “It [the incompleteness result] is closely related to the “Liar”” . But it is wrong to say that Gödel´s argument is equivalent to the Liar, because Gödel used a slightly different statement for his proof. The reason for this slight shift is, that the Liar´s confession renders a contradiction, whereas the statement which was used by Gödel in his proof is not contradictory, although it looks as if it is. Gödel´s statement runs as follows:

This statement cannot be proved

It is of utmost importance to see the point: that the Liar´s confession leads to a contradiction, whereas this statement does not!

2. Mathematicians become emotional when they comment on Gödel´s result

Now let me, before I present the kernel of Wittgenstein´s reaction to Gödel, embark on the reactions of mathematician to this important contribution to mathematics. There is an important author, whom some of you may already know : I am talking of Stuart Shanker. He contributes a paper of more than a hundred pages on „Wittgenstein and Gödel“. There are two important points, which I want to quote to you.

One of the most important facts in this long paper is that Shanker compares Gödel´s result with a symphony (1988 page 156). (He quotes the idea from Nagel&Newman 1958 page 94f).

It is quite remarkable that a philosopher of mathematics should become poetical about a mathematical result; one could even say that Shanker becomes emotional about this result of the year 1931. There is a strong positive connotation in his evaluation. There is a rule in the philosophy of science that one should not make value judgements about scientific results - yet that is exactly what Shanker does: he expresses his high esteem of Gödel´s theorems without shyness. The direction of his statement is a positive one: you will see in a moment why I have to stress this.

There are, as well, negative statements about Gödel – for example in „Collier´s encyclopaedia“. I quote from the edition of the year 1969. In an article on the philosophy of mathematics, it is said that Gödel has „unfortunately“ (1969 Vol 15, page 550a) proved his result. It is quite an unprecedented expression in the philosophy of mathematics to say that a result has been achieved „unfortunately“. I want to engrave this on your minds and this is why I want to repeat myself: the anonymous author within this encylcopaedia says that Gödel has „unfortunately“ proved this result.

So here is my conclusion in this section: there is a contradiction about Gödel. Some authors are quite delighted about the ingenuity of Gödel´s proof – and others are disgusted by it.

3. Wittgenstein sees the Liar´s Paradox as a useless language game

This contradiction on the level of attitudes towards Gödel warns us that an investigation on the reception of this result is not such an easy task.

I have found a quotation in an essay by a mathematician who is in error about Wittgenstein and Gödel: he states (wrongly) that Wittgenstein never made a single remark about Gödel. Karl Sigmund writes: "Another remarkable parallel between Hahn and Wittgenstein is that both never mentioned Gödel in their philosophical writings" (DePauli-Schimanovich/Köhler/Stadler 1995 page 240). This statement annoys me: Wittgenstein has, on the contrary, done quite a lot of work on Gödel, his LFM (VGM in German), which is in the long version called „Lectures on the Foundations of Mathematics Cambridge 1939“ are mainly devoted to Gödel.

In this book Wittgenstein makes a remark, which is central to our topic. Wittgenstein's position is as following:

> "„I am lying“ [...] it is just a useless language game and why should anybody be excited" (Wittgenstein LFM Unit XXI page 207)

Indeed, Wittgenstein´s statement accurately reflects the attitude of mathematicians of around 1900 towards our problem. Some researchers knew very well about the contradictions which arise when one mathematicizes the known paradoxes. Especially Cantor did know that there were „inconsistent sets“, like the well known set of all sets.

The problem with Wittgenstein´s statement is that it is dis-integrative; the Liar´s paradox is excluded from mathematics, and here Wittgenstein works right against Gödel, whose ingenious idea was, on the contrary, to mathematicize the liar´s paradox by using this Gödel numbering technique. This had beforehand seemed to be impossible.

4. To give up the solving of a puzzle

My fourth paragraph concerns puzzles, especially unsolvable puzzles. This paragraph is a central point in my presentation; now when I talk on puzzles, I could also chose the word „enigma“, which means essentially the same. It is the Greek word and the reason why I shall use it is, that a puzzle might be confused with a jigsaw puzzle. A jigsaw puzzle contains a collection of pieces made from cardboard. It is for children and it has a remarkable property: if one has enough time and motivation, then a jigsaw puzzle always has a solution. (Unless if a part of cardboard has been lost.) When I talk about enigmata, then I want to use a concept which arises for example, when one reads Thomas Samuel Kuhn.

Here, it is not known whether the enigma has a solution or not. Here, when I discuss this property hopefully in a clear manner, then there is an acute logical distinction: there are enigmata that have a solution and there are enigmata that do not.

When one studies the history of science, whereby the history of mathematics is meant here, a lot of fuss is made about such puzzles – Kuhn uses the word “crisis”. Kuhn´s point is: ”Failure to achieve a solution discredits only the scientist and not the theory” (1996 page 80). If one wants to be a normal scientist, it is advisable not to tackle unsolvable puzzles.

Wittgenstein´s central point in his LFM is the following: if several researchers try to solve the enigma and many researchers fail to be successful, this does not mean that the puzzle is unsolvable. Wittgenstein uses the word "we" which is a short form of what Kuhn later calls the scientific community: "We ... perhaps gave up the problem altogether" (LFM Unit IX page 88) It might even be the case that some researchers suspect that the puzzle may be unsolvable, while others are still entangled in fruitless research. Those who are suspicious will stop participating in research, but as long as they cannot frame their suspicious attitude into a proof, the others will continue in their search.

5. The question of a hidden contradiction

Wittgenstein´s critique of Gödel´s results of 1931 must be subdivided into his reception of Gödel´s 1st Incompleteness Theorem and his reception of Gödel´s 2nd Incompleteness Theorem. What I have said about unsolvable puzzles refers to the first incompleteness theorem – now let us embark on the second. The question of hidden contradictions arises here.

The problem here is the principle of "ex falso sequitur quodlibet": from a contradictory proposition anything can be concluded. It is exactly this point that makes Gödel´s second Incompleteness Theorem a little bit confusing: If mathematics should contain a contradiction, then it can be proved that mathematics is free from contradictions!

Wittgenstein has a simple solution for this "horror contradictionis" from which mathematicians suffer: he says "Well then, just don´t draw any conclusions from a contradiction".(LFM Unit XXII page 220). Now I want to refer to Collier´s Encyclopaedia again and we shall see that mathematicians have adopted Wittgenstein´s method:

> "... no contradiction has ever been detected during that period. So most mathematicians have simply stopped worrying about these matters and go on with their work as if they believed that no contradiction will ever occur" (Vol 15 page 550 column a). So, we can conclude from this statement that mathematicians just shrug their shoulders and – they largely ignore Gödel´s result.

6. Gödel´s reaction to Wittgenstein´s reaction to the Incompleteness Theorems

Now I must warn you, this paragraph contains a frustrating bit of literature. When it comes to research work done on Gödel, Hao Wang is an important author: his book, "Reflections on Kurt Gödel", is a voluminous account of the talks which he had with Gödel. The miracle which becomes apparent here is, that Gödel allowed Hao Wang to come into contact with him. On page 48, under the title "Relation to the Schlick Circle", Hao Wang begins to report on Wittgenstein. Wittgenstein occurs relatively often in this book – from the Index, it can be seen, that he occurs more than 40 times.

Hao Wang shows that Gödel studied Wittgenstein´s Tractatus in 1927, and the later reading of Wittgenstein was presented to him by Hao Wang. Hao Wang and Gödel discussed Wittgenstein on the 5th of April 1972, but the material which Hao Wang produces in his book in enriched by a letter which Gödel sent to the mathematician Menger on the 20th of May in the same year. I want to present the quotation from Hao Wangs book in full, because it is so important. Here, Gödel writes:

> "As far as my theorem about undecidable propositions is concerned, it is indeed clear ... that Wittgenstein did *not* understand it (or pretended not to understand it). He interprets it as a kind of logical paradox, while in fact it is just the opposite, namely a mathematical theorem within an absolutely uncontroversial part on mathematics (finitary number theory or combinatorics)" (page 49).

When I first encountered this quotation about two years ago, this was a severe attack on my personal comfort – and even when I read it now, it is torture for me.

Now I would like to bring in Thomas Samuel Kuhn – in his famous essay he says that when revolutions occur, then the researchers among each other produce many misunderstandings. What I do think about Gödel´s statement is the following: Gödel said that Wittgenstein did not understand him, but did Gödel understand Wittgenstein?? I think that the question of whether Wittgenstein has understood Gödel is controversial in character and the side which I want to take here is, that Wittgenstein did understand Gödel, though in a creative manner "in einer geisteswissenschaftlichen Art" (as a part of the humanities).

7. Wittgensteins Remarks on Gödel´s Results

Wittgenstein´s "Remarks on the Foundations of Mathematics" were published in 1956, but then in 1978 an enlarged edition was published. I want to quote from section VII of this edition, most of which was written by Wittgenstein in January of the year 1941 (see page 31 for this point). Although Wittgenstein´s remark on Gödel does not contain the appropriate respect, which Gödel would have deserved, but it is a very clear statement, which produces in bewilderment on the part of the logicians. Wittgenstein notes:

> "My task is, not to talk about ... Gödel´s proof, but to by-pass it." (1978 page 383) This statement is not very polite, and it is not entirely clear what Wittgenstein is heading for when he articulates it. I would like to give an interpretation, but I am not very firm here: I am weak. Wittgenstein should have been more serious about Gödel´s results.

Gödel´s proof of 1931 is a firm mathematical result – there do not arise any questions about it. It is an utterly uncontroversial result and Wittgenstein rather should accept it than to make unclear statements about it. When, about a year ago, I wanted to quote this passage from memory I paraphrased it in the following way: Wittgenstein said, that we should *ignore* Gödel´s result. This expression “ignore” is definitely too strong to be a correct interpretation of what Wittgenstein really says.

But Gödel´s proof is not only a mathematical result. It is also a philosophical result, which circumscribes mathematics from the outside. One can approach – and I think, this is what Wittgenstein intends to do – Gödel´s result from a standpoint which lies within the humanities. Wittgenstein wants to look at Gödel´s proof not as a mathematical result, but as a part of the humanities ("Geisteswissenschaften") . This is what he means with his statement about bypassing Gödel.

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